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The Hall effect and bend resistance in ballistic quantum wires constructed out of quantum dots

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Abstract. We present a tight-binding theory of the four-terminal resistances of junctions of single-mode ballistic quantum wires constructed out of quantum dots. The Hall and bend resistances depend critically on the *direct* coupling between quantum dots in *different* quantum wires. We predict that under some conditions such systems should exhibit quenching of the Hall effect as well as negative Hall resistances due to quantum interference. The structures that we discuss are realizable experimentally.

In the last few years, experiments on quasi-ballistic one-dimensional (1D) conductors in semiconductor heterostructures have revealed a number of intriguing transport phenomena associated with their junctions. These include the Roukes effect (the disappearance or 'quenching' of the Hall voltage across a 1D conductor at low magnetic fields) [1], the local and non-local bend resistances of Takagaki *et al* [2] and Timp *et al* [3], and the negative Hall resistance of Ford *et al* [4]. These experiments have stimulated considerable theoretical interest in the behaviour of electrons at the junctions of 1D conductors [5]. In this article we report on the first theoretical study of the Hall effect in junctions of 1D conductors of a new type–ballistic conductors in the form of periodic chains of quantum dots. Arrays of quantum dots have been fabricated in semiconductor heterostructures by a variety of techniques which confine electrons of a 2D electron gas to regions about 1000 Å in diameter [6]. The feasibility of making a 1D ballistic conductor from a chain of quantum dots has been discussed theoretically by Ulloa *et al* [7], and very recently such structures have been realized experimentally by Kouwenhoven *et al* and Haug *et al* [8]. The system that we consider is in the single-mode quantum regime, which is not the case in the usual 1D conductors [1–4]. However, we predict that it should also exhibit the interesting quenching of the Hall effect and negative Hall resistance, although the physical mechanism is different. The single-mode theory should be applicable to small quantum dots, a few tens of nanometres in radius in GaAs. In this case, the strong confinement results in a sufficiently large splitting between the lowest energy level that we consider here and higher levels at magnetic fields up to about 10 T [9].

Our model is shown schematically in figure 1, where the quantum dots are represented by shaded circles. There is a magnetic field B applied perpendicular to the x–y plane containing the quantum dots. The four quantum-dot leads are connected to electron reservoirs at electro-chemical potentials μ_i, $i = 1, 2, 3, 4$. There is a net current of electrons I_i in lead i. The voltage of the ith reservoir is given by $V_i = \mu_i/e$, where e is

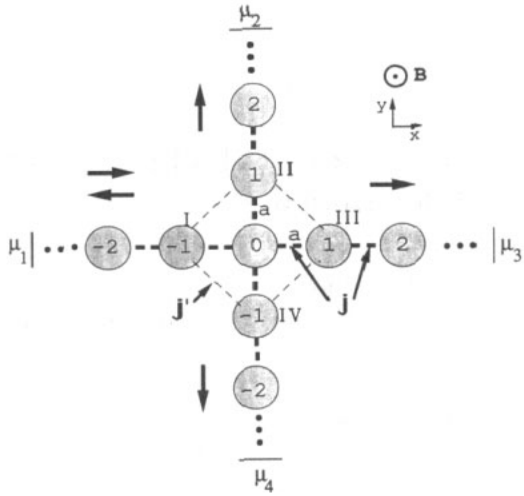

Figure 1. The model of a junction of quantum wires formed from quantum dots.

the electron charge. In the usual four-terminal Hall measurement, two leads, say $i = 1$ and 3, carry currents. This leads to the condition $I_1 = -I_3 = I$. The other two leads are voltage leads and do not carry any net current, $I_2 = I_4 = 0$. For the longitudinal resistance measurements, $I_1 = -I_3 = I$, $I_2 = I_4 = 0$ as for the Hall resistance. We also calculate the bend resistance that corresponds to the condition of $I_1 = -I_4 = I$ and $I_2 = I_3 = 0$. Using these current conditions, the Hall, longitudinal and bend resistances R_{4H}, R_{4L} and R_{4B} are defined in terms of V_i and I_i as follows:

$$R_{4H} = (\mu_2 - \mu_4)/Ie \qquad R_{4L} = (\mu_1 - \mu_3)/Ie \qquad R_{4B} = (\mu_2 - \mu_3)/Ie. \tag{1}$$

The currents in the four leads can be obtained from the single-mode Büttiker equations [10]:

$$I_i = \frac{e}{h}\left(\mu_i - \sum_j T_{ij}\mu_j\right) \qquad i, j = 1, 2, 3, 4 \tag{2}$$

where T_{ij} is the probability of an electron incident on the junction in lead j being scattered into lead i and T_{ii} is the reflection coefficient for lead i. We ignore spin and calculate the T_{ij} using the tight-binding Hamiltonian:

$$H = \sum_{\alpha\beta} J_{\alpha\beta} |\varphi_\alpha\rangle\langle\varphi_\beta| \tag{3}$$

where $|\varphi_\alpha\rangle$ is the tight-binding state of the αth quantum dot. Here we assume that each quantum dot has only one level, i.e. the quantum wires are single mode. In the presence of an applied magnetic field, the transfer integrals $J_{\alpha\beta}$ are modified according to [10]

$$J_{\alpha\beta} \to J_{\alpha\beta} \exp[\mathrm{i}(e/2\hbar)\boldsymbol{B}\cdot(\boldsymbol{r}_\alpha \times \boldsymbol{r}_\beta)] \tag{4}$$

where $\boldsymbol{r}_\alpha$ is the position vector of the αth quantum dot and we choose the origin of coordinates in figure 1 at the central quantum dot (labelled 0). Suppose an electron with wave vector k coming from reservoir 1 is incident on the junction. The wave function will be scattered into the other three leads 2, 3, 4 and reflected partially back to reservoir

1. In the tight-binding model, the wave function is written as $|\Psi\rangle = \Sigma_n c_n|\varphi_n\rangle$ where $|\varphi_n\rangle$ is the quantum state in the nth quantum dot and c_n is the wave amplitude on the nth dot. C_n has the following form for the four leads: $c_n = e^{ikan} + t_{11}e^{-ikan}$ $(x < 0)$, $c_n = t_{31}e^{ikan}$ $(x > 0)$, $c_n = t_{21}e^{ikan}$ $(y > 0)$ and $c_n = t_{41}e^{-ikan}$ $(y < 0)$, where the distance between two adjacent dots in the 1D chains is assumed to be a, t_{i1} is the scattering amplitude into ith lead and t_{11} is the reflection amplitude back into lead 1.

Consider to start with the simplest case where only the transfer integrals between nearest-neighbour quantum dots in the 1D chains in figure 1 are non-zero ($J_{\alpha\beta} = j$). In this case the phase factors in $\exp[i(e/2\hbar)\boldsymbol{B}\cdot(\boldsymbol{r}_\alpha \times \boldsymbol{r}_\beta)]$ in (4) are all equal to unity and the magnetic field does not appear in the Hamiltonian (3). As a consequence R_H and R_B are independent of $\boldsymbol{B}$, and since for a symmetric junction in this case $T_{21} = T_{31} = T_{41}$, R_H and R_B are both identically zero in the nearest-neighbour tight-binding approximation. For the tight-binding Hamiltonian (3), we find that the reflection and transmission probabilities, $R = T_{11}$ and $T = T_{31} = T_{21} = T_{41}$, are $R = 1/[4\sin(ka)^2 + \cos(ka)^2]$ and $T = \sin(ka)^2/[4\sin(ka)^2 + \cos(ka)^2]$. Thus the longitudinal resistance obtained from the Büttiker formula (2) reduces in this case to:

$$R_{4L} = (h/2e^2)/T = (h/2e^2)(16 - 3\varepsilon^2)/(4 - \varepsilon^2) \tag{5}$$

where $\varepsilon = E/j = 2\cos(ka)$ is the normalized band energy of the Fermi electrons, E is the Fermi energy and k is the Fermi wave vector. Figure 2 shows the relationship between R_{4L} and ε. The longitudinal resistance is almost constant and approximately equal to $2h/e^2$, except close to the band edges, where there are strong junction reflections.

Thus in order to describe the effects of a magnetic field and to calculate the Hall and bend resistance in tight binding theory, one should include the next nearest-neighbour transfer integrals j' between quantum dots in different chains, as shown in figure 1. According to (4), the j' acquire a phase in a magnetic field; $j' \to j'e(\pm\frac{1}{2}i\pi\,\phi/\phi_0)$ where the $+$ $(-)$ corresponds to counter-clockwise (clockwise) transfers relative to the central quantum dot 0. $\phi = 2a^2B$ is the magnetic flux through the square whose corners are the dots labelled I, II, III, IV and $\phi_0 = h/e$ is the elementary flux quantum. Thus the total phase change around the smallest closed loop in the system is $\frac{1}{4}(2\pi\phi/\phi_0) = \frac{1}{4}\tau$, where τ is the total phase change around the largest loop.

Thus from (3) we obtain the equations for transmission and reflection coefficients:

$$\begin{aligned}
&j(r_{11}+1) - ja_0 = j'e^{ika}(t_{21}e^{-i\tau/4} + t_{41}e^{i\tau/4}) \\
&jt_{21} - ja_0 = j'e^{ika}[t_{31}e^{-i\tau/4} + (r_{11} + e^{-2ika})e^{i\tau/4}] \\
&jt_{31} - ja_0 = j'e^{ika}(t_{21}e^{i\tau/4} + e^{-i\tau/4}t_{41}) \\
&jt_{41} - ja_0 = j'e^{ika}[t_{31}e^{i\tau/4} + (r_{11} + e^{-2ika})\,e^{-i\tau/4}] \\
&Ea_0 = j[e^{-ika} + e^{ika}(r_{11} + t_{21} + t_{31} + t_{41})]
\end{aligned} \tag{6}$$

where a_0 is the wave function amplitude at the central dot 0 of figure 1, and $E = 2j\cos(ka)$. The transmission and reflection probabilities that depend on magnetic field and Fermi energy ε are obtained from (4).

For symmetric junctions, the following magneto-resistance formulae follow from (2) [5]:

$$\begin{aligned}
&R_{4H} = (h/e^2)(T_{21} - T_{41})/Z \qquad R_{4L} = (h/e^2)(T_{21} + T_{41} + 2T_{31})/Z \\
&R_{4B} = (h/e^2)(T_{31}^2 - T_{21}T_{41})/Z'
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
&Z = T_{21}^2 + T_{41}^2 + 2T_{31}(T_{31} + T_{21} + T_{41}) \\
&Z' = (T_{21} + T_{41})[(T_{31} + T_{21})^2 + (T_{31} + T_{41})^2].
\end{aligned}$$

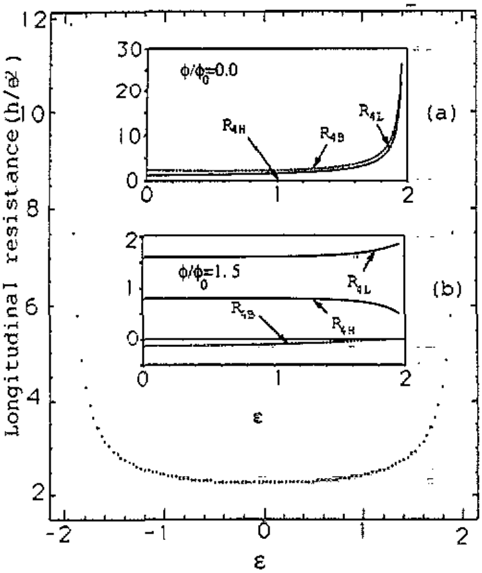

Figure 2. R_{4L} against Fermi energy ε with $j' = 0$. Insets a and b show R_{4H}, R_{4L} and R_{4B} as functions of Fermi energy ε at $j'/j = 0.7$ for $\phi/\phi_0 = 0$ and $\phi/\phi_0 = 1.5$ respectively.

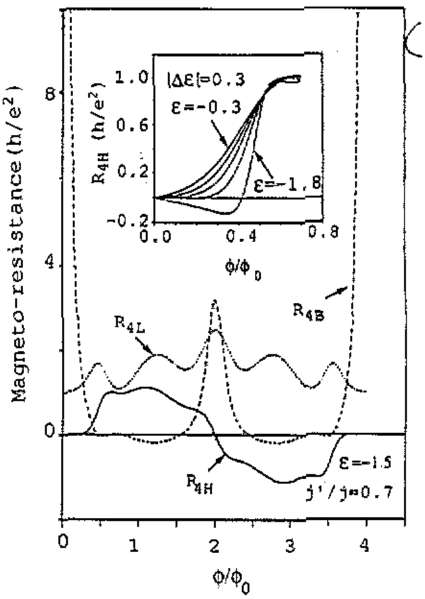

Figure 3. R_{4H}, R_{4L} and R_{4B} against magnetic field ϕ/ϕ_0 at $j'/j = 0.7$, $\varepsilon = -1.5$. The insets shows the variation of R_{4H} against ϕ/ϕ_0 at five different Fermi energies.

In the present model, the resistances R_{4H}, R_{4L} and R_{4B} depend on only three variables: the normalized Fermi energy $\varepsilon = E/j = 2\cos(ka)$, which is common to all four reservoirs; the dimensionless magnetic field $\phi/\phi_0 = 2a^2Be/h$; and the relative value of the transfer integrals j'/j. For comparison with the case of $j' = 0$, insets a and b in figure 2 show R_{4H}, R_{4L} and R_{4B} as a function of ε at two fixed magnetic fields $\phi/\phi_0 = 0$ and $\phi/\phi_0 = 1.5$ with $j'/j = 0.7$. It is apparent that R_{4H} and R_{4B} are no longer identically equal to zero if the next-nearest-neighbour transfer integral j' is included. The results [12] for R_{4H}, R_{4L} and R_{4B} against magnetic field at $j'/j = 0.7$ and $\varepsilon = -1.5$ are shown in figure 3; they are all periodic in magnetic field with period four in units of ϕ/ϕ_0, corresponding to one magnetic flux quantum change through the smallest closed loop. Clearly this periodicity derives from Aharonov–Bohm-type interference. If we remove the central quantum dot from the junction (see inset in figure 4), the magnetic flux through the smallest closed loop is four times larger and we should expect that the oscillation period should reduce to one in units of ϕ/ϕ_0. This can be seen clearly in figure 4, where all three resistances have period one. In addition it is clear from figure 3 that R_{4L} and R_{4B} are symmetric and that R_{4H} is anti-symmetric about $\phi/\phi_0 = 2$ at fixed Fermi energy ε. That is, $R_{4L,B}(2 - \phi/\phi_0, \varepsilon = R_{4L,B}(2 + \phi/\phi_0, \varepsilon)$ and $R_{4H}(2 - \phi/\phi_0, \varepsilon) = -R_{4H}(2 + \phi/\phi_0, \varepsilon)$, which derives from the interchange of phase factors of the transfer integrals $j'_{1\to 2}$ and $j'_{1\to 4}$ when we make a symmetry reflection about $\phi/\phi_0 = 2$. Another interesting effect that we find is that the Hall resistance quenches for certain magnetic field values, for some values of the Fermi energy and j'/j (see inset in figure 3). The quench found here comes from a mechanism which is completely different from that in the experiments on conventional 1D conductors [1–4] which has been the subject of many theoretical papers [5]. Here it is a result of Aharonov–Bohm-type phase interference.

The above treatment is for a junction of infinite chains of quantum dots. In reality

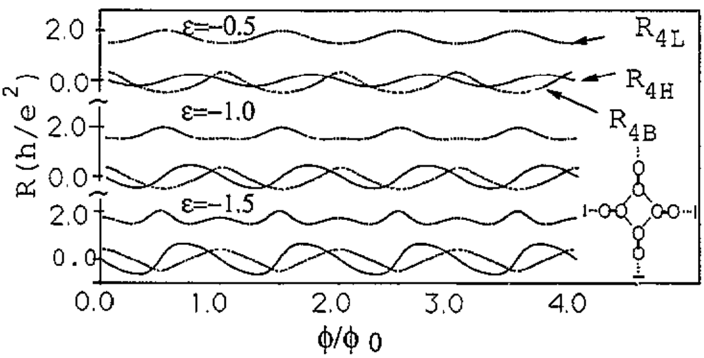

Figure 4. R_{4H}, R_{4L} and R_{4B} against magnetic field for the structure without central quantum dot. Note $R_{4H} = 0$ for $\varepsilon = 0$ in this model.

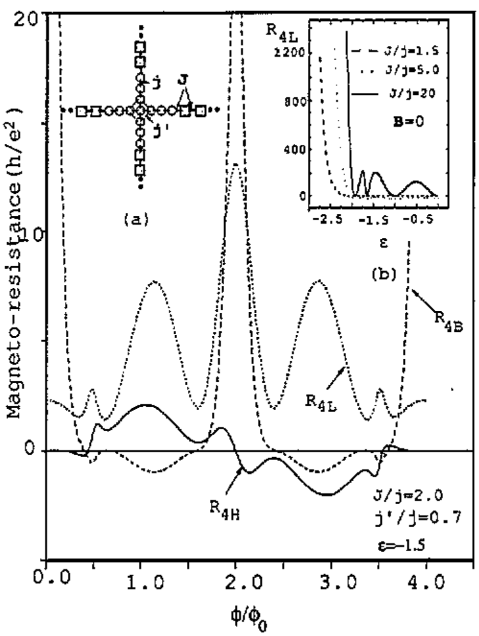

Figure 5. R_{4H}, R_{4L} and R_{4B} against magnetic field at $J/j = 2$, $j'/j = 0.7$ and $\varepsilon = -1.5$. Inset a shows the geometry of model and inset b shows R_{4L} at $B = 0$ against ε for three different values of J/j.

finite systems of quantum dots will be coupled to reservoirs by leads consisting of a free electron gas, and the matching between the quantum dot chains and these leads needs to be considered. Within the tight-binding framework, the effect of this matching can be studied in a simplified way by modelling the free electron gas leads by chains of quantum dots with a wider 1D energy band and correspondingly larger transfer integral J than that of the dots forming the junction, as shown in inset a of figure 5. In this case the four-terminal resistance will also depend on J/j, which determines the degree of matching between the 1D quantum dot chains and electron gas leads, as well as on j'/j. Figure 5 shows the Hall, longitudinal and bend resistances as a function of magnetic field at $\varepsilon = 1.5$, $J/j = 2$ and $j'/j = 0.7$, for the structure in inset a (the squares are dots that model free electron gas leads); except J, the parameters are the same as in figure 3. Inset b of figure 5 shows the dependence of the longitudinal resistance on the Fermi energy ε at magnetic field $\boldsymbol{B} = 0$ for different transfer integral ratios $J/j = 1.5$, 5.0 and 20.

Comparison of figures 3 and 5 shows that the mismatching does not change the behaviour of the three resistances qualitatively well inside the energy range between $2j$ and $-2j$, but affects the relationship between resistance and magnetic field drastically if the energy is outside of this range. Physially the reason for this is the fact that there is a strong reflection of incident electrons if their energy is outside of or near the edge of the energy band of the quantum dots forming the junction. Reflection is dominant in that latter range while the Aharonov–Bohm phase effect is dominant in the former.

In conclusion, we have presented the behaviour of the four-terminal resistances of junctions of infinite and finite 1D quantum dot chains in the tight-binding approximation. The Lorentz force in the usual sense does not exist in the present model, because it is strongly non-classical [11b]. Thus, the phase modification of the electron wave function induced by the magnetic field plays a crucial role in the Hall effect. As a result, we anticipate that the Hall resistance will quench at some magnetic fields for certain Fermi energies.

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